Artham
Resource Material

For Revised Syllabus Session 2024-25

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Chapter 5-
Complex Numbers And Quadratic Equations

EXERCISE

SHORT ANSWER TYPE

1. **For a positive integer n, find the value of $(1 - i)^n (1 - 1/i)^n$.**

Solution:

According to the question,

We have,

$$(1-i)^n\left(1-\frac{1}{i}\right)^n = (1-i)^n\left(1-\frac{1}{i^2}\right)^n$$

$= (1 - i)^n (1 + i)^n$

$= (1 - i^2)^n$

$= 2^n$

Therefore, $(1 - i)^n (1 - 1/i)^n = 2^n$

2. Evaluate

$$\sum_{n=1}^{13}\left(i^n + i^{n+1}\right),$$

where n ϵ N.

Solution:

According to the question,

We have,

$$\sum_{n=1}^{13}(i^n + i^{n+1}) = \sum_{n=1}^{13}(1+i)i^n$$

$$= (1 + i)\,(1 + i^2 + i^3 + i^4 + i^5 + i^6 + i^7 + i^8 + i^9 + i^{10} + i^{11} + i^{12} + i^{13})$$

$$= (1+i)\frac{i(i^{13}-1)}{i-1}$$

$$= (1+i)\frac{i(i-1)}{i-1}$$

$$= (1 + i)\,i$$

$$= i + i^2$$

$$= i - 1$$

$$\therefore \sum_{n=1}^{13}(i^n + i^{n+1}) = i - 1$$

3. If

$$\left(\frac{1+i}{1-i}\right)^3 - \left(\frac{1-i}{1+i}\right)^3 = x + iy$$

, then find (x, y).

Solution:

According to the question,

We have,

$$x + iy = \left(\frac{1+i}{1-i}\right)^3 - \left(\frac{1-i}{1+i}\right)^3$$

$$= \left(\frac{(1+i)^2}{1-i^2}\right)^3 - \left(\frac{(1-i)^2}{1-i^2}\right)^3$$

$$= \left(\frac{1+2i+i^2}{1+1}\right)^3 - \left(\frac{1-2i+i^2}{1+1}\right)^3$$

$$= \left(\frac{2i}{2}\right)^3 - \left(\frac{-2i}{2}\right)^3$$

$= i^3 - (-i^3)$

$= 2i^3$

$= 0 - 2i$

Thus, $(x, y) = (0, -2)$

4. If $\frac{(1+i)^2}{2-i} = x + iy$ **, then find the value of x + y.**

Solution:

According to the question,

We have,

$$x + iy = \frac{(1+i)^2}{2-i}$$

$$= \frac{1+2i+i^2}{2-i}$$

$$= \frac{2i}{2-i}$$

Rationalizing the denominator,

$$= \frac{2i(2+i)}{(2-i)(2+i)}$$

$$= \frac{4i+2i^2}{4-i^2}$$

$$= \frac{4i-2}{4+1}$$

$$= \frac{-2}{5} + \frac{4i}{5}$$

Thus,

$$x = -\frac{2}{5}, y = \frac{4}{5}$$

Hence,

$$x + y = -\frac{2}{5} + \frac{4}{5} = \frac{2}{5}$$

5. If

$$\left(\frac{1-i}{1+i}\right)^{100} = a + ib;$$ **then find (a, b).**

Solution:

According to the question,

We have,

$$a + ib = \left(\frac{1-i}{1+i}\right)^{100}$$

$$= \left[\frac{1-i}{1+i}.\frac{1-i}{1-i}\right]^{100}$$

$$= \left[\frac{(1-i)^2}{1-i^2}\right]^{100}$$

$$= \left(\frac{1-2i+i^2}{1+1}\right)^{100}$$

$$= \left(-\frac{2i}{2}\right)^{100}$$

$= (i^4)^{25}$

$= 1$

Hence, (a, b) = (1, 0)

6. **If a = cos θ + i sin θ, find the value of**

$$\frac{1+a}{1-a}.$$

Solution:

According to the question,

We have,

$a = \cos\theta + i\sin\theta$

$$\Rightarrow \frac{1+a}{1-a} = \frac{(1+\cos\theta) + i\sin\theta}{(1-\cos\theta) - i\sin\theta}$$

$$= \frac{2\cos^2\frac{\theta}{2} + i2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\sin^2\frac{\theta}{2} - i2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}$$

$$= \frac{2\cos\frac{\theta}{2}\left(\cos\frac{\theta}{2} + i\sin\frac{\theta}{2}\right)}{2\sin\frac{\theta}{2}\left(\sin\frac{\theta}{2} - i\cos\frac{\theta}{2}\right)}$$

$$= \frac{i\cos\frac{\theta}{2}\left(\cos\frac{\theta}{2} + i\sin\frac{\theta}{2}\right)}{\sin\frac{\theta}{2}\left(i\sin\frac{\theta}{2} - i^2\cos\frac{\theta}{2}\right)}$$

$$= \frac{i\cos\frac{\theta}{2}\left(\cos\frac{\theta}{2} + i\sin\frac{\theta}{2}\right)}{\sin\frac{\theta}{2}\left(i\sin\frac{\theta}{2} + \cos\frac{\theta}{2}\right)}$$

$$= i\cot\frac{\theta}{2}$$

7. If $(1 + i)z = (1 – i)\bar{z}$, then show that $z = i\bar{z}$.

Solution:

According to the question,

We have,

$$(1 + i)z = (1 - i)\bar{z}$$

$$\Rightarrow z = \frac{1 - i}{1 + i}\bar{z}$$

Rationalizing the denominator,

We get,

$$= \frac{(1 - i)(1 - i)}{(1 + i)(1 - i)}\bar{z}$$

$$= \frac{(1 - i)^2}{(1 - i^2)}\bar{z}$$

$$= \frac{1 - 2i + i^2}{1 + 1}\bar{z}$$

$$= \frac{1 - 2i - 1}{2}\bar{z}$$

$= -i\bar{z}$

Hence proved.

8. If $z = x + iy$, then show that $z\bar{z} + 2(z + \bar{z}) + b = 0$ where $b \epsilon R$, representing z in the complex plane is a circle.

Solution:

According to the question,

We have,

$z = x + iy$

$\Rightarrow \bar{z} = x – iy$

Now, we also have,

$z\bar{z} + 2(z + \bar{z}) + b = 0$

$\Rightarrow (x + iy)(x – iy) + 2(x + iy + x – iy) + b = 0$

$\Rightarrow x^2 + y^2 + 4x + b = 0$

The equation obtained represents the equation of a circle.

9. If the real part of $(\bar{z} + 2)/(\bar{z} – 1)$ is 4, then show that the locus of the point representing z in the complex plane is a circle.

Solution:

According to the question,

Let $z = x + iy$

Now,

$$\frac{\bar{z}+2}{\bar{z}-1}=\frac{x-iy+2}{x-iy-1}$$

$$=\frac{[(x+2)-iy][(x-1)+iy]}{[(x-1)-iy][(x-1)+iy]}$$

$$=\frac{(x-1)(x+2)+y^2+i[(x+2)y-(x-1)y]}{(x-1)^2+y^2}$$

According to the question, we have, real part = 4.

$$\Rightarrow \frac{(x-1)(x+2)+y^2}{(x-1)^2+y^2}=4$$

$\Rightarrow x^2 + x - 2 + y^2 = 4\,(x^2 - 2x + 1 + y^2)$

$\Rightarrow 3x^2 + 3y^2 - 9x + 6 = 0$

The equation obtained represents the equation of a circle.

Hence, locus of z is a circle.

10. Show that the complex number z, satisfying the condition arg ((z-1)/(z+1)) = π/4 lies on a circle.

Solution:

According to the question,

Let $z = x + iy$

$\arg((z-1)/(z+1)) = \pi/4$

$\Rightarrow \arg(z-1) - \arg(z+1) = \pi/4$

$\Rightarrow \arg(x + iy - 1) - \arg(x + iy + 1) = \pi/4$

$\Rightarrow \arg(x - 1 + iy) - \arg(x + 1 + iy) = \pi/4$

$$\Rightarrow \tan^{-1}\frac{y}{x-1}+\tan^{-1}\frac{y}{x+1}=\frac{\pi}{4}$$

$$\Rightarrow \tan^{-1}\left[\frac{\frac{y}{x-1}-\frac{y}{x+1}}{1+\left(\frac{y}{x-1}\right)\left(\frac{y}{x+1}\right)}\right]=\frac{\pi}{4}$$

$$\Rightarrow \frac{y(x+1-x+1)}{x^2-1+y^2}=\tan\frac{\pi}{4}$$

$$\Rightarrow \frac{2y}{x^2+y^2-1}=1$$

$\Rightarrow x^2 + y^2 - 1 = 2y$

$\Rightarrow x^2 + y^2 - 2y - 1 = 0$

The equation obtained represents the equation of a circle.

11. **Solve that equation |z| = z + 1 + 2i.**

Solution:

According to the question,

We have,

$|z| = z + 1 + 2i$

Substituting $z = x + iy$, we get,

$\Rightarrow |x + iy| = x + iy + 1 + 2i$

We know that,
$|z| = \sqrt{(x^2 + y^2)}$
$\sqrt{(x^2 + y^2)} = (x + 1) + i(y + 2)$
Comparing real and imaginary parts,
We get,
$\sqrt{(x^2 + y^2)} = (x + 1)$
And $0 = y + 2$
$\Rightarrow y = -2$
Substituting the value of y in $\sqrt{(x^2 + y^2)} = (x + 1)$,
We get,
$\Rightarrow x^2 + (-2)^2 = (x + 1)^2$
$\Rightarrow x^2 + 4 = x^2 + 2x + 1$
Hence, $x = 3/2$
Hence, $z = x + iy$
$= 3/2 – 2i$

LONG ANSWER TYPE

12. If $|z + 1| = z + 2 (1 + i)$, then find z.
Solution:

According to the question,
We have,
$|z + 1| = z + 2 (1 + i)$
Substituting $z = x + iy$, we get,
$\Rightarrow |x + iy + 1| = x + iy + 2 (1 + i)$
We know,
$|z| = \sqrt{(x^2 + y^2)}$
$\sqrt{((x + 1)^2 + y^2)} = (x + 2) + i(y + 1)$
Comparing real and imaginary parts,
$\Rightarrow \sqrt{((x + 1)^2 + y^2)} = x + 2$
And $0 = y + 2$
$\Rightarrow y = -2$
Substituting the value of y in $\sqrt{((x + 1)^2 + y^2)} = x + 2$,
$\Rightarrow (x + 1)^2 + (-2)^2 = (x + 2)^2$
$\Rightarrow x^2 + 2x + 1 + 4 = x^2 + 4x + 4$
$\Rightarrow 2x = 1$
Hence, $x = ½$
Hence, $z = x + iy$
$= ½ – 2i$

13. If $\arg (z – 1) = \arg (z + 3i)$, then find $x – 1 : y$. where $z = x + iy$
Solution:

According to the question,
Let $z = x + iy$
Given that,
$\arg (z – 1) = \arg (z + 3i)$
$\Rightarrow \arg (x + iy – 1) = \arg (x + iy + 3i)$

⇒ arg (x – 1 + iy) = arg (x + I (y) = π/4

$$\Rightarrow \tan^{-1}\frac{y}{x-1} = \tan^{-1}\frac{y+3}{x}$$

$$\Rightarrow \frac{y}{x-1} = \frac{y+3}{x}$$

⇒ $xy = xy - y + 3x - 3$

⇒ $3x - 3 = y$

⇒ $(x - 1)/y = 1/3$

Hence, $(x - 1): y = 1: 3$

14. Show that |(z – 2) / (z – 3)| = 2 represents a circle. Find its centre and radius.

Solution:

According to the question,

We have,

$|(z - 2) / (z - 3)| = 2$

Substituting $z = x + iy$, we get

⇒ $|(x + iy - 2) / (x + iy - 3)| = 2$

⇒ $|x - 2 + iy| = 2\,|x - 3 + iy|$

⇒ $\sqrt{((x - 2)^2 + y^2)} = 2\sqrt{((x - 3)^2 + y^2)}$

⇒ $x^2 - 4x + 4 + y^2 = 4\,(x^2 - 6x + 9 + y^2)$

⇒ $3x^2 + 3y^2 - 20x + 32 = 0$

$$\Rightarrow x^2 + y^2 - \frac{20}{3}x + \frac{32}{3} = 0$$

$$\Rightarrow \left(x - \frac{10}{3}\right)^2 + y^2 + \frac{32}{3} - \frac{100}{9} = 0$$

$$\Rightarrow \left(x - \frac{10}{3}\right)^2 + (y - 0)^2 = \frac{4}{9}$$

Therefore, centre of circle is (10/3, 0) and radius is 4/9 or 2/3.

15. If (z – 1)/(z + 1) is a purely imaginary number (z ≠ –1), then find the value of |z|.

Solution:

According to the question,

Let $z = x + iy$

Now,

$$\frac{z-1}{z+1} = \frac{x+iy-1}{x+iy+1}$$

$$= \frac{[(x-1)+iy][(x+1)-iy]}{[(x+1)+iy][(x+1)-iy]}$$

$$= \frac{(x^2-1)+y^2+i[(x+1)y-(x-1)y]}{(x+1)^2+y^2}$$

According to the question, we have,

$\frac{z-1}{z+1}$ is purely imaginary.

$$\Rightarrow \frac{(x^2-1)+y^2}{(x+1)^2+y^2} = 0$$

$\Rightarrow x^2 - 1 + y^2 = 0$
$\Rightarrow x^2 + y^2 = 1$
$\Rightarrow \sqrt{(x^2 + y^2)} = 1$
Hence, $|z| = 1$

16. z_1 and z_2 are two complex numbers such that $|z_1| = |z_2|$ and $\arg(z_1) + \arg(z_2) = \pi$, then show that $z_1 = -\bar{z}_2$.

Solution:

According to the question,
Let $z_1 = |z_1| (\cos \theta_1 + I \sin \theta_1)$ and $z_2 = |z_2| (\cos \theta_2 + I \sin \theta_2)$
Given that $|z_1| = |z_2|$
And $\arg (z_1) + \arg (z_2) = \pi$
$\Rightarrow \theta_1 + \theta_2 = \pi$
$\Rightarrow \theta_1 = \pi - \theta_2$
Now, $z_1 = |z_2| (\cos (\pi - \theta_2) + I \sin (\pi - \theta_2))$
$\Rightarrow z_1 = |z_2| (-\cos \theta_2 + I \sin \theta_2)$
$\Rightarrow z_1 = -|z_2| (\cos \theta_2 - I \sin \theta_2)$
$\Rightarrow z_1 = -[|z_2| (\cos \theta_2 - I \sin \theta_2)]$
Hence, $z_1 = -\bar{z}_2$
Hence proved.

17. If $|z_1| = 1$ ($z_1 \neq -1$) and $z_2 = (z_1 - 1) / (z + 1)$, then show that the real part of z_2 is zero.

Solution:

According to the question,
Let $z_1 = x + iy$

$$\Rightarrow |z_1| = \sqrt{x^2 + y^2} = 1$$

$$z_2 = \frac{z_1 - 1}{z_1 + 1} = \frac{x + iy - 1}{x + iy + 1}$$

$$= \frac{[(x-1) + iy][(x+1) - iy]}{[(x+1) + iy][(x+1) - iy]}$$

$$= \frac{(x^2 - 1) + y^2 + i[(x+1)y - (x-1)y]}{(x+1)^2 + y^2}$$

$$= \frac{x^2 + y^2 - 1 + 2iy}{(x+1)^2 + y^2}$$

Since $x^2 + y^2 = 1$

$$= \frac{1 - 1 + 2iy}{(x+1)^2 + y^2}$$

$$= 0 + \frac{2iy}{(x+1)^2 + y^2}$$

Therefore, the real part of z_2 is zero.

18. If z_1, z_2 and z_3, z_4 are two pairs of conjugate complex numbers, then find arg(z_1/z_4) + arg(z_2/z_3).

Solution:

According to the question,

We have,

z_1 and z_2 are conjugate complex numbers.

The negative side of the real axis

$= r_1 (\cos \theta_1 - i \sin \theta_1)$

$= r_1 [\cos (-\theta_1) + I \sin (-\theta_1)]$

Similarly, $z_3 = r_2 (\cos \theta_2 - i \sin \theta_2)$

$\Rightarrow z_4 = r_2 [\cos (-\theta_2) + I \sin (-\theta_2)]$

$$\Rightarrow \arg\left(\frac{z_1}{z_4}\right) + \arg\left(\frac{z_2}{z_3}\right) = \arg(z_1) - \arg(z_4) + \arg(z_2) - \arg(z_3)$$

$= \theta_1 - (-\theta_2) + (-\theta_1) - \theta_2$

$= \theta_1 + \theta_2 - \theta_1 - \theta_2$

$= 0$

$\Rightarrow \arg(z_1/z_4) + \arg(z_2/z_3) = 0$

19. If $|z_1| = |z_2| = \ldots.. = |z_n| = 1$, then show that $|z_1 + z_2 + z_3 + \ldots. + z_n| = |\, 1/z_1 + 1/z_2 + 1/z_3 + \ldots + 1/z_n|$

Solution:

According to the question,

We have,

$|z_1| = |z_2| = \ldots = |z_n| = 1$

$\Rightarrow |z_1|^2 = |z_2|^2 = \ldots = |z_n|^2 = 1$

$\Rightarrow z_1 \overline{z}_1 = z_2 \overline{z}_2 = z_3 \overline{z}_3 = \ldots = z_n \overline{z}_n = 1$

$$\Rightarrow z_1 = \frac{1}{\overline{z_1}}, z_2 = \frac{1}{\overline{z_2}}, \dots, z_n = \frac{1}{\overline{z_n}}$$

Now,

$$\Rightarrow |z_1 + z_2 + z_3 + z_4 + \dots + z_n| = \left|\frac{z_1\overline{z_1}}{\overline{z_1}} + \frac{z_2\overline{z_2}}{\overline{z_2}} + \frac{z_3\overline{z_3}}{\overline{z_3}} + \dots + \frac{z_n\overline{z_n}}{\overline{z_n}}\right|$$

$$= \left|\frac{1}{\overline{z_1}} + \frac{1}{\overline{z_2}} + \frac{1}{\overline{z_3}} + \dots + \frac{1}{\overline{z_n}}\right|$$

$$= \left|\overline{\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3} + \dots + \frac{1}{z_n}}\right|$$

$$= \left|\frac{1}{z_1} + \frac{1}{z_2} + \dots + \frac{1}{z_n}\right|$$

Hence proved.

20. If for complex numbers z_1 and z_2, arg (z_1) – arg (z_2) = 0, then show that $|z_1 – z_2| = |z_1| – |z_2|$.

Solution:

According to the question,

Let $z_1 = |z_1| (\cos \theta_1 + I \sin \theta_1)$ and $z_2 = |z_2| (\cos \theta_2 + I \sin \theta_2)$

We have,

arg (z_1) - arg (z_2) = 0

$\Rightarrow \theta_1 - \theta_2 = 0$

$\Rightarrow \theta_1 = \theta_2$

We also have,

$z_2 = |z_2| (\cos \theta_1 + I \sin \theta_1)$

$\Rightarrow z_1 – z_2 = ((|z_1|\cos \theta_1 - |z_2| \cos \theta_1) + i (|z_1| \sin \theta_1 - |z_2| \sin \theta_1))$

$$\Rightarrow |z_1 - z_2| = \sqrt{(|z_1|\cos\theta_1 - |z_2|\cos\theta_1)^2 + (|z_1|\sin\theta_1 - |z_2|\sin\theta_1)^2}$$

$$= \sqrt{|z_1|^2 + |z_2|^2 - 2|z_1||z_2|\cos^2\theta_1 - 2|z_1||z_2|\sin^2\theta_1}$$

$$= \sqrt{|z_1|^2 + |z_2|^2 - 2|z_1||z_2|[\cos^2\theta_1 + \sin^2\theta_1]}$$

We know that $\cos^2 \theta + \sin^2 \theta = 1$

$$= \sqrt{|z_1|^2 + |z_2|^2 - 2|z_1||z_2|}$$

$$= \sqrt{(|z_1| - |z_2|)^2}$$

Hence, $|z_1 – z_2| = |z_1| - |z_2|$

Hence proved.

21. Solve the system of equations Re (z_2) = 0, |z| = 2.

Solution:

According to the question,

We have,

Re (z^2) = 0, |z| = 2

Let z = x + iy.

Then, $|z| = \sqrt{(x^2 + y^2)}$

Given in the question,

$\sqrt{(x^2 + y^2)} = 2$

$\Rightarrow x^2 + y^2 = 4 \ldots$ (i)
$z^2 = x^2 + 2ixy - y^2$
$= (x^2 - y^2) + 2ixy$
Now, Re $(z^2) = 0$
$\Rightarrow x^2 - y^2 = 0 \ldots$ (ii)
Equating (i) and (ii), we get
$\Rightarrow x^2 = y^2 = 2$
$\Rightarrow x = y = \pm\sqrt{2}$
Hence, $z = x + iy$
$= \pm\sqrt{2} \pm i\sqrt{2}$
$= \sqrt{2} + i\sqrt{2}$, $\sqrt{2} - i\sqrt{2}$, $-\sqrt{2} + i\sqrt{2}$ and $-\sqrt{2} - i\sqrt{2}$
Hence, we have four complex numbers.

22. Find the complex number satisfying the equation $z + \sqrt{2}\,|(z + 1)| + i = 0$.
Solution:

According to the question,
We have,
$z + \sqrt{2}\,|(z + 1)| + i = 0 \ldots (1)$
Substituting $z = x + iy$, we get
$\Rightarrow x + iy + \sqrt{2}\,|x + iy + 1| + i = 0$
$\Rightarrow x + i(1 + y) + \sqrt{2}\left[\sqrt{(x+1)^2 + y^2}\right] = 0$
$\Rightarrow x + i(1 + y) + \sqrt{2}\sqrt{(x^2 + 2x + 1 + y^2)} = 0$
Comparing real and imaginary parts to zero, we get
$\Rightarrow x + \sqrt{2}\sqrt{x^2 + 2x + 1 + y^2} = 0 \ldots (2)$
And,
$y + 1 = 0$
$\Rightarrow y = -1$
Substituting $y = -1$ into equation (2), we get
$\Rightarrow x + \sqrt{2}\sqrt{x^2 + 2x + 1 + 1} = 0$
$\Rightarrow \sqrt{2}\sqrt{x^2 + 2x + 2} = -x$
$\Rightarrow 2x^2 + 4x + 4 = x^2$
$\Rightarrow x^2 + 4x + 4 = 0$
$\Rightarrow (x + 2)^2 = 0$
$\Rightarrow x = -2$
Hence, $z = x + iy$
$= -2 - i$

23. Write the complex number

$$z = \frac{1 - i}{\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}}$$

in polar from.

Solution:

According to the question,
We have,

$$z = \frac{1-i}{\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}}$$

$$= \frac{\sqrt{2}\left[\frac{1}{\sqrt{2}} - i\frac{1}{\sqrt{2}}\right]}{\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}}$$

$$= \frac{\sqrt{2}\left[\cos\frac{\pi}{4} - i\sin\frac{\pi}{4}\right]}{\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}}$$

$$= \sqrt{2}\left[\cos\left(-\frac{\pi}{4} - \frac{\pi}{3}\right) + i\sin\left(-\frac{\pi}{4} - \frac{\pi}{3}\right)\right]$$

$$= \sqrt{2}\left[\cos\left(-\frac{7\pi}{12}\right) + i\sin\left(-\frac{7\pi}{12}\right)\right]$$

$$= \sqrt{2}\left[\cos\frac{5\pi}{12} + i\sin\frac{5\pi}{12}\right]$$

24. If z and w are two complex numbers such that |zw| = 1 and arg (z) – arg (w) = π/2, then show that $\bar{z}$w = – i.

Solution:

Let z = |z| (cos θ_1 + I sin θ_1) and w = |w| (cos θ_2 + I sin θ_2)
Given |zw| = |z| |w| = 1
Also arg (z) – arg (w) = π/2
⇒ θ_1 - θ_2 = π/2
Now, $\bar{z}$ w = |z| (cos θ_1 - I sin θ_1) |w| (cos θ_2 + I sin θ_2)g | = 1
= |z| |w| (cos (-θ_1) + I sin (-θ_1)) (cos θ_2 + I sin θ_2)
= 1 [cos (θ_2 – θ_1) + I sin (θ_2 – θ_1)]
= [cos (-π/2) + I sin (-π/2)]
= 1 [0 – i]
= – i
Hence proved.

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- **Timely Updates and Reminders:** SOE WhatsApp groups serve as a source of timely updates and reminders about important dates, exam schedules, syllabus changes, and academic events. Teachers can stay informed and well-prepared for upcoming assessments and activities.

- **Interactive Learning Environment:** Teachers can engage in discussions, ask questions, and seek clarifications within the group, creating an interactive learning environment. This fosters collaboration, peer learning, and knowledge sharing among group members, enhancing understanding and retention of concepts.

- **Access to Expert Guidance:** SOE WhatsApp groups are moderated by subject matter experts, teachers, or experienced educators can benefit from their guidance, expertise, and insights on various academic topics, exam strategies, and study techniques.

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Best Regards,
Team
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Kindergarten to Class XII (For Teachers Only)

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Class 10

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Secondary Groups (IX & X)

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Hindi-B

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Senior Secondary Groups (XI & XII For Teachers Only)

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Other Important Groups (For Teachers & Principal's)

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Kindergarten to Class XII (For Students Only)

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Class 11 (Humanities)

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Class 12 (Humanities)

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Chemistry

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AI

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IIT/NEET

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CUET

Groups Rules & Regulations:

To maximize the benefits of these WhatsApp groups, follow these guidelines:

1. Share your valuable resources with the group.
2. Help your fellow educators by answering their queries.
3. Watch and engage with shared videos in the group.
4. Distribute WhatsApp group resources among your students.
5. Encourage your colleagues to join these groups.

Additional notes:

1. Avoid posting messages between 9 PM and 7 AM.
2. After sharing resources with students, consider deleting outdated data if necessary.
3. It's a NO Nuisance groups, single nuisance and you will be removed.
    - No introductions.
    - No greetings or wish messages.
    - No personal chats or messages.
    - No spam. Or voice calls
    - Share and seek learning resources only.

Please only share and request learning resources. For assistance, contact the helpline via WhatsApp: +91-95208-77777.

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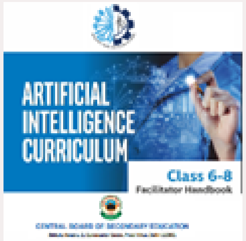
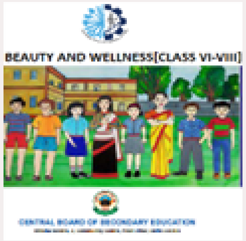
School of Educators & Artham Resources

SKILL MODULES BEING OFFERED IN MIDDLE SCHOOL


ARTIFICIAL INTELLIGENCE CURRICULUM
Class 6-8
Facilitator Handbook
CENTRAL BOARD OF SECONDARY EDUCATION

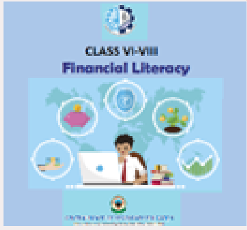
Artificial Intelligence


BEAUTY AND WELLNESS[CLASS VI-VIII]
CENTRAL BOARD OF SECONDARY EDUCATION

Beauty & Wellness

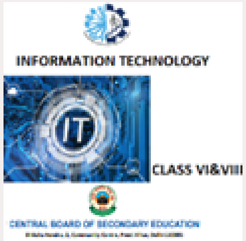
DESIGN THINKING&INNOVATION
CLASS VI-VIII
CENTRAL BOARD OF SECONDARY EDUCATION

Design Thinking & Innovation



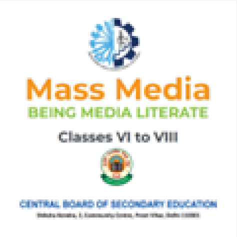
Financial Literacy

Handicrafts



Information Technology

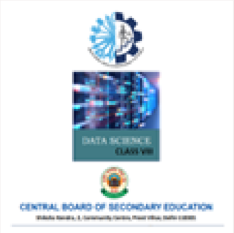
Marketing/Commercial Application



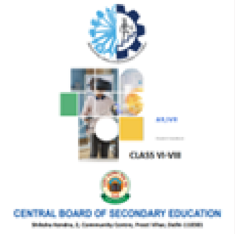
Mass Media - Being Media Literate

Travel & Tourism

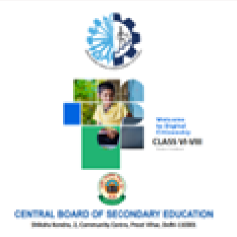
Coding



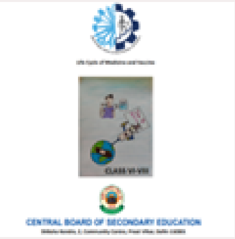
Data Science (Class VIII only)



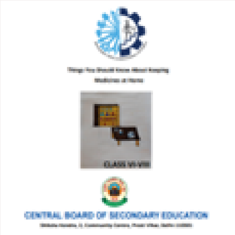
Augmented Reality / Virtual Reality



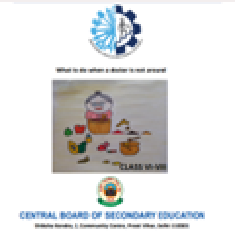
Digital Citizenship



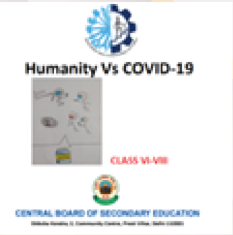
Life Cycle of Medicine & Vaccine



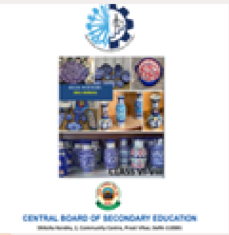
Things you should know about keeping Medicines at home



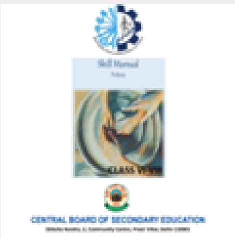
What to do when Doctor is not around



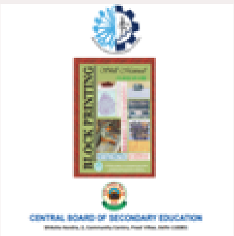
Humanity & Covid-19



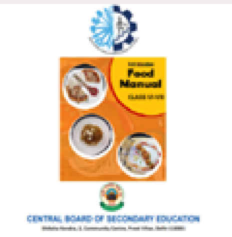
Blue Pottery



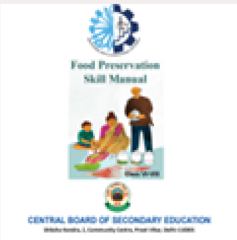
Pottery



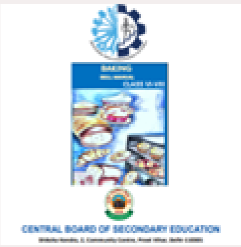
Block Printing



Food

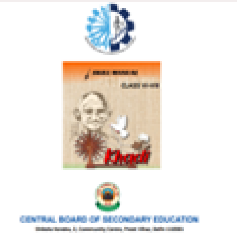


Food Preservation

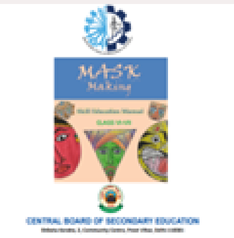


Baking

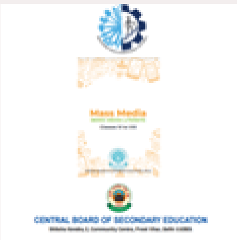
Herbal Heritage



Khadi

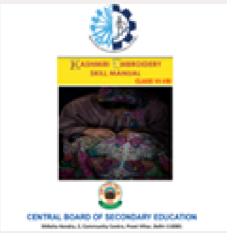


Mask Making



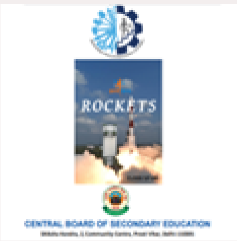
Mass Media

Making of a Graphic Novel

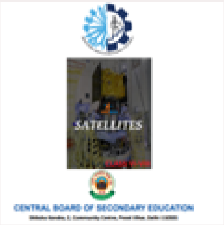


Kashmiri Embroidery

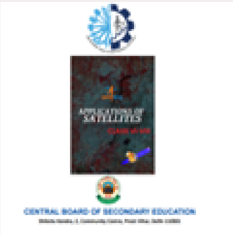
Embroidery



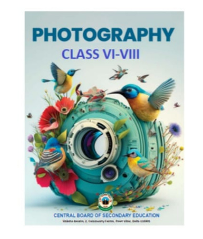
Rockets



Satellites

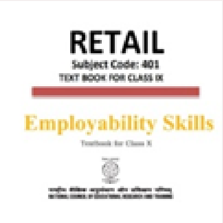


Application of Satellites

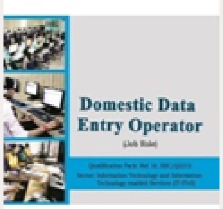


Photography

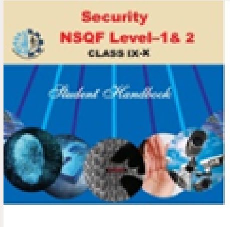
SKILL SUBJECTS AT SECONDARY LEVEL (CLASSES IX – X)


RETAIL
Subject Code: 401
TEXT BOOK FOR CLASS IX
Employability Skills
Textbook for Class X

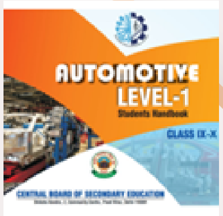
Retail


Domestic Data Entry Operator
(Job Role)

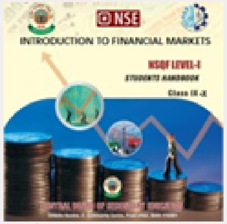
Information Technology


Security
NSQF Level-1& 2
CLASS IX-X
Student Handbook

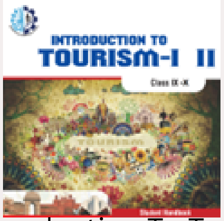
Security


AUTOMOTIVE
LEVEL-1
Students Handbook
CLASS IX-X
CENTRAL BOARD OF SECONDARY EDUCATION

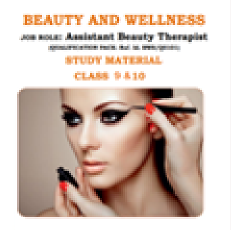
Automotive


NSE
INTRODUCTION TO FINANCIAL MARKETS
NSQF LEVEL-I
STUDENTS HANDBOOK
Class IX-X

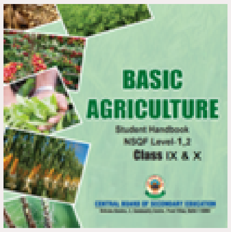
Introduction To Financial Markets


INTRODUCTION TO
TOURISM-I II
Class IX-X

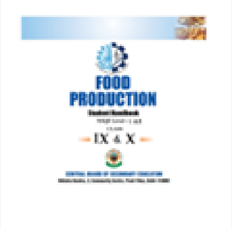
Introduction To Tourism


BEAUTY AND WELLNESS
JOB ROLE: Assistant Beauty Therapist
STUDY MATERIAL
CLASS 9 & 10

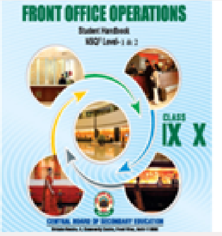
Beauty & Wellness


BASIC AGRICULTURE
Student Handbook
NSQF Level-1,2
Class IX & X
CENTRAL BOARD OF SECONDARY EDUCATION

Agriculture


FOOD PRODUCTION
Student Handbook
CLASS
IX & X
CENTRAL BOARD OF SECONDARY EDUCATION

Food Production


FRONT OFFICE OPERATIONS
Student Handbook
NSQF Level-1 & 2
CLASS
IX X
CENTRAL BOARD OF SECONDARY EDUCATION

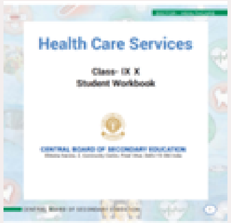
Front Office Operations

Banking & Insurance
Study Material
Standard IX & X
Student Handbook
CENTRAL BOARD OF SECONDARY EDUCATION

Banking & Insurance

MARKETING
AND
SALES
Study Material
IX & X
Student Handbook
CENTRAL BOARD OF SECONDARY EDUCATION

Marketing & Sales


Health Care Services
Class- IX X
Student Workbook
CENTRAL BOARD OF SECONDARY EDUCATION

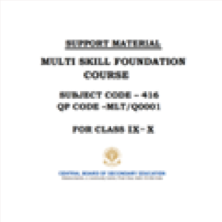
Health Care

Apparel
Hand Embroiderer
(Job Role)

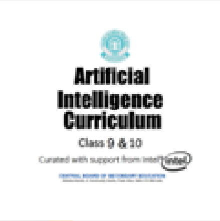
Apparel

STUDY MATERIAL
MEDIA (415)
CLASS - IX & X
Media & Entertainment Skills Council
CENTRAL BOARD OF SECONDARY EDUCATION

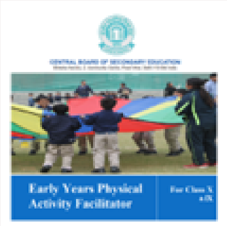
Multi Media


SUPPORT MATERIAL
MULTI SKILL FOUNDATION COURSE
SUBJECT CODE - 416
QP CODE -MLT/Q0001
FOR CLASS IX - X
CENTRAL BOARD OF SECONDARY EDUCATION

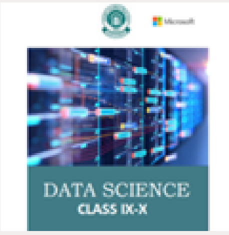
Multi Skill Foundation Course


Artificial Intelligence Curriculum
Class 9 & 10
Curated with support from Intel
CENTRAL BOARD OF SECONDARY EDUCATION

Artificial Intelligence


CENTRAL BOARD OF SECONDARY EDUCATION
Early Years Physical Activity Facilitator
For Class X & IX

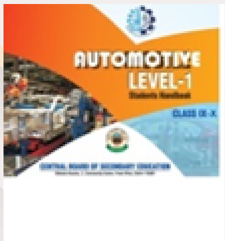
Physical Activity Trainer


Microsoft
DATA SCIENCE
CLASS IX-X

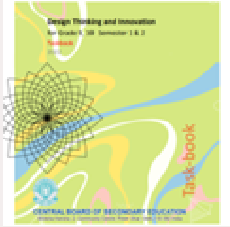
Data Science

CENTRAL BOARD OF SECONDARY EDUCATION
JOB ROLE: FIELD TECHNICIAN - OTHER HOME APPLIANCES
STUDENT TEXTBOOK
CLASS 9 & 10
Sector: ELECTRONICS AND HARDWARE

Electronics & Hardware (NEW)

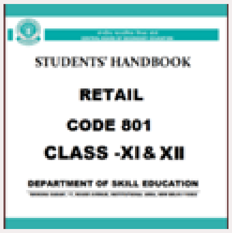

AUTOMOTIVE
LEVEL-1
Students Handbook
CLASS IX-X
CENTRAL BOARD OF SECONDARY EDUCATION

Foundation Skills For Sciences (Pharmaceutical & Biotechnology)(NEW)

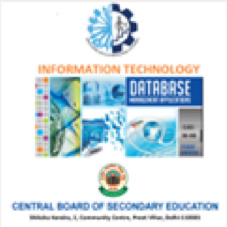

Design Thinking and Innovation
Task book
CENTRAL BOARD OF SECONDARY EDUCATION

Design Thinking & Innovation (NEW)

SKILL SUBJECTS AT SR. SEC. LEVEL (CLASSES XI – XII)


STUDENTS' HANDBOOK
RETAIL
CODE 801
CLASS -XI & XII
DEPARTMENT OF SKILL EDUCATION

Retail


INFORMATION TECHNOLOGY
DATABASE
CENTRAL BOARD OF SECONDARY EDUCATION

InformationTechnology

Web Applications
Students Handbook
CLASS
XI-XII
CENTRAL BOARD OF SECONDARY EDUCATION

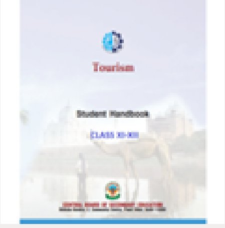
Web Application

AUTOMOTIVE
CLASS XI-XII
CENTRAL BOARD OF SECONDARY EDUCATION

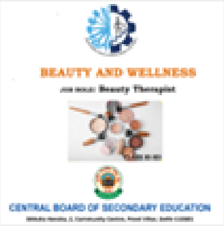
Automotive

FINANCIAL MARKETS MANAGEMENT
CLASS XI-XII
CENTRAL BOARD OF SECONDARY EDUCATION

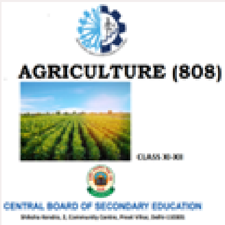
Financial Markets Management


Tourism
Student Handbook
CLASS XI-XII

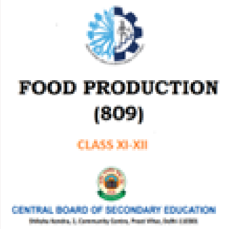
Tourism


BEAUTY AND WELLNESS
Beauty Therapist
CENTRAL BOARD OF SECONDARY EDUCATION

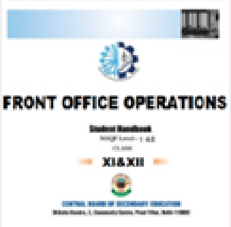
Beauty & Wellness


AGRICULTURE (808)
CLASS XI-XII
CENTRAL BOARD OF SECONDARY EDUCATION

Agriculture


FOOD PRODUCTION
(809)
CLASS XI-XII
CENTRAL BOARD OF SECONDARY EDUCATION

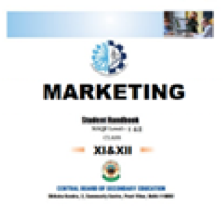
Food Production


FRONT OFFICE OPERATIONS
Student Handbook
XI&XII

Front Office Operations

BANKING
Student Handbook
XI&XII

Banking


MARKETING
Student Handbook
XI&XII

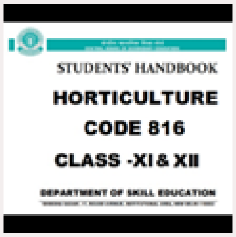
Marketing

SECTOR : HEALTHCARE
Student Workbook
CLASS XI&XII
Central Board of Secondary Education

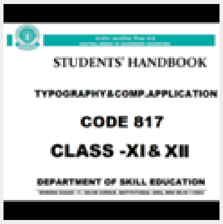
Health Care

INSURANCE
Student Handbook
XI&XII

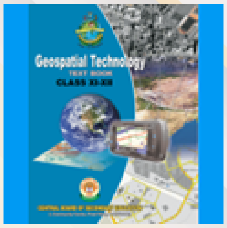
Insurance


STUDENTS' HANDBOOK
HORTICULTURE
CODE 816
CLASS -XI & XII
DEPARTMENT OF SKILL EDUCATION

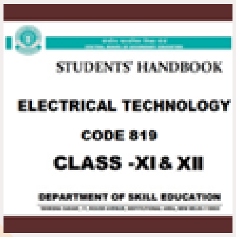
Horticulture


STUDENTS' HANDBOOK
TYPOGRAPHY&COMP.APPLICATION
CODE 817
CLASS -XI & XII
DEPARTMENT OF SKILL EDUCATION

Typography & Comp. Application


Geospatial Technology
TEXT BOOK
CLASS XI-XII

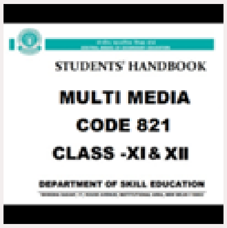
Geospatial Technology


STUDENTS' HANDBOOK
ELECTRICAL TECHNOLOGY
CODE 819
CLASS -XI & XII
DEPARTMENT OF SKILL EDUCATION

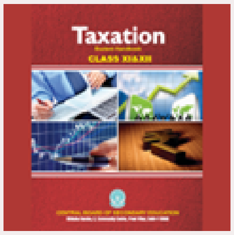
Electrical Technology

BASIC ELECTRONICS
Student Handbook
CLASS XI&XII

Electronic Technology


STUDENTS' HANDBOOK
MULTI MEDIA
CODE 821
CLASS -XI & XII
DEPARTMENT OF SKILL EDUCATION

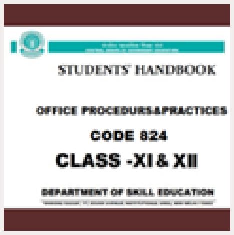
Multi-Media


Taxation
CLASS XI&XII

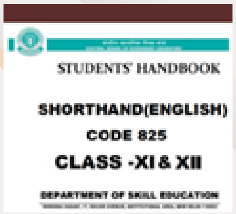
Taxation

Cost Accounting
CLASS XI&XII

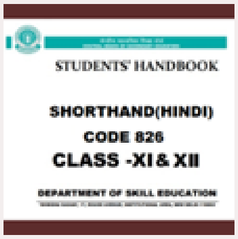
Cost Accounting


STUDENTS' HANDBOOK
OFFICE PROCEDURES&PRACTICES
CODE 824
CLASS -XI & XII
DEPARTMENT OF SKILL EDUCATION

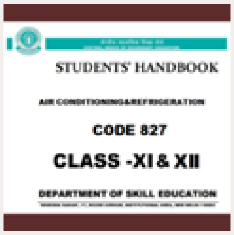
Office Procedures & Practices


STUDENTS' HANDBOOK
SHORTHAND(ENGLISH)
CODE 825
CLASS -XI & XII
DEPARTMENT OF SKILL EDUCATION

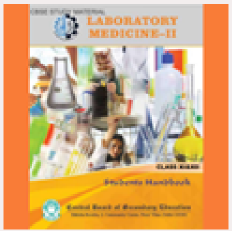
Shorthand (English)


STUDENTS' HANDBOOK
SHORTHAND(HINDI)
CODE 826
CLASS -XI & XII
DEPARTMENT OF SKILL EDUCATION

Shorthand (Hindi)


STUDENTS' HANDBOOK
AIR CONDITIONING&REFRIGERATION
CODE 827
CLASS -XI & XII
DEPARTMENT OF SKILL EDUCATION

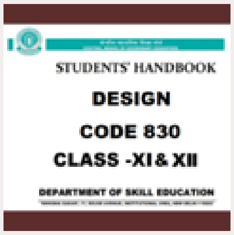
Air-Conditioning & Refrigeration


LABORATORY MEDICINE-II
CLASS XI&XII
Students Handbook
Central Board of Secondary Education

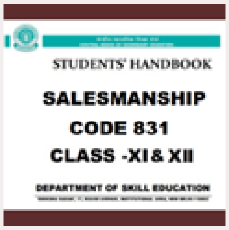
Medical Diagnostics

TEXTILE SCIENCE
Text Book &
Practical Manual

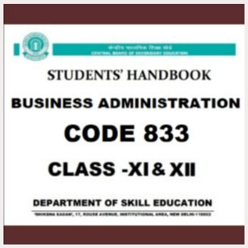
Textile Design


STUDENTS' HANDBOOK
DESIGN
CODE 830
CLASS -XI & XII
DEPARTMENT OF SKILL EDUCATION

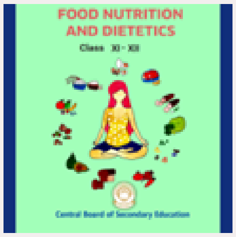
Design


STUDENTS' HANDBOOK
SALESMANSHIP
CODE 831
CLASS -XI & XII
DEPARTMENT OF SKILL EDUCATION

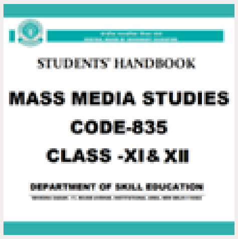
Salesmanship


STUDENTS' HANDBOOK
BUSINESS ADMINISTRATION
CODE 833
CLASS -XI & XII
DEPARTMENT OF SKILL EDUCATION

Business Administration


FOOD NUTRITION AND DIETETICS
Class XI - XII
Central Board of Secondary Education

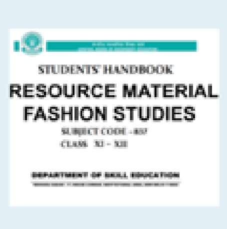
Food Nutrition & Dietetics


STUDENTS' HANDBOOK
MASS MEDIA STUDIES
CODE-835
CLASS -XI & XII
DEPARTMENT OF SKILL EDUCATION

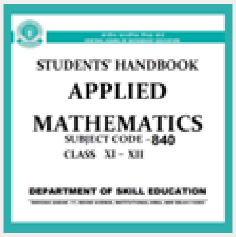
Mass Media Studies

CENTRAL BOARD OF SECONDARY EDUCATION
LIBRARY AND INFORMATION SCIENCE
CLASS XI-XII

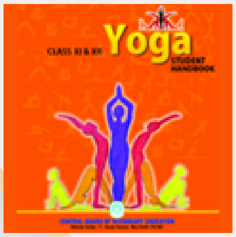
Library & Information Science


STUDENTS' HANDBOOK
RESOURCE MATERIAL
FASHION STUDIES
SUBJECT CODE - 837
CLASS XI - XII
DEPARTMENT OF SKILL EDUCATION

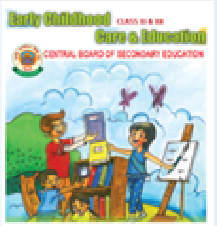
Fashion Studies


STUDENTS' HANDBOOK
APPLIED
MATHEMATICS
SUBJECT CODE - 840
CLASS XI - XII
DEPARTMENT OF SKILL EDUCATION

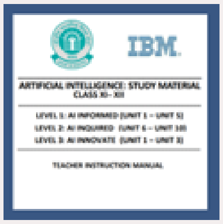
Applied Mathematics


CLASS XI & XII
Yoga
STUDENT HANDBOOK

Yoga


Early Childhood Care & Education
CLASS XI & XII
CENTRAL BOARD OF SECONDARY EDUCATION

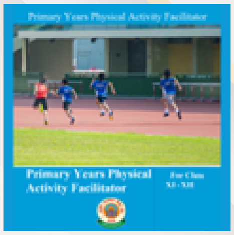
Early Childhood Care & Education


IBM.
ARTIFICIAL INTELLIGENCE: STUDY MATERIAL
CLASS XI – XII
LEVEL 1: AI INFORMED (UNIT 1 – UNIT 5)
LEVEL 2: AI INQUIRED (UNIT 6 – UNIT 10)
LEVEL 3: AI INNOVATE (UNIT 1 – UNIT 3)
TEACHER INSTRUCTION MANUAL

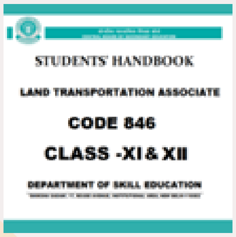
Artificial Intelligence

DATA SCIENCE
CLASS XI-XII

Data Science


Primary Years Physical Activity Facilitator
Primary Years Physical Activity Facilitator
For Class XI - XII

Physical Activity Trainer(new)


STUDENTS' HANDBOOK
LAND TRANSPORTATION ASSOCIATE
CODE 846
CLASS -XI & XII
DEPARTMENT OF SKILL EDUCATION

Land Transportation Associate (NEW)

Installation Technician
Computing and Peripherals
Sector: Electronics and Hardware
Vocational Education
CENTRAL BOARD OF SECONDARY EDUCATION

Electronics & Hardware (NEW)

Design Thinking and Innovation
Task-book

Design Thinking & Innovation (NEW)